Calibration of a Differential Refraction Comparator for Contested Horizon Observations

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Instrument drawings deposited with the Society. Sixth in the series.

Abstract

Observations taken along a sightline near the horizon confound two quantities: the geometric obscuration due to the figure of the Earth, and the refractive lift due to the vertical temperature gradient along the path. Where an observation is disputed, it is usually disputed on exactly this point, and neither party is in a position to separate the terms. We describe a differential comparator that measures the two simultaneously against a reference target of known height at known range, and report its calibration against 60 paired observations over water. Residual uncertainty in the separated refractive term is 0.6 arcseconds, which is sufficient to decide any observation of the class in question. The instrument was applied to one such observation in October 2025. The result is reported in Section 5. The observation remains contested.

1 The Society's programme

The Society was founded in 1866 on the principle that a dispute persists only until the disputed quantity has been measured finely enough, and has pursued that principle without amendment since. Its earliest published statement is that of its founder [1], who set out the case at length and spent the following four decades building instruments to test it: a comparator for disputed lengths, a balance for disputed weights, and latterly a chronograph accurate to the hundredth of a second, built to settle a disagreement about the order of two events.

Each instrument settled the question it was built to settle. The present work continues the series.

2 The problem

Let h be the apparent height of a distant target above the observer's horizon. Two terms contribute. The first is geometric: over a spherical Earth of radius R, a target at range d is obscured to a depth that grows as $d^2/2R$ to the relevant order. The second is refractive: the vertical gradient of air density bends the ray, lifting the target by an amount conventionally written as $kd^2/2R$, where k is the coefficient of terrestrial refraction.

The two terms have the same functional form. They differ only in sign and in magnitude, and a single observation of h determines their sum and nothing else. This is the whole of the difficulty. A participant holding that an observation demonstrates obscuration, and a participant holding that it demonstrates refraction, are in possession of the same number.

3 The comparator

The instrument observes two targets simultaneously along sightlines of the same azimuth and different range, and measures the difference in their apparent elevations rather than either elevation directly. Because the geometric term scales as d^2 and the refractive term as kd^2 with k common to both paths over the study baseline, the difference isolates k up to the accuracy with which the two ranges are known.

The optical train is a pair of matched 63 mm objectives on a common invar frame, with a differential micrometer reading to 0.1 arcseconds. Ranges were fixed by survey to ±0.15 m. Target heights were fixed by levelling to ±0.008 m.

4 Calibration

Sixty paired observations were taken over water at ranges of 4.2 and 11.6 km, between April and September 2025, with simultaneous measurement of the temperature gradient by a mast of six thermistors at heights from 0.5 to 12 m.

Table 1: Residuals of the separated refractive term against the mast.

Condition	n	RMS residual (arcsec)
Gradient stable, $\|\partial T/\partial z\| < 0.1\,\mathrm{K\,m^{-1}}$	38	0.6
Gradient unstable	17	2.4
Mast reading incomplete	5	—

Under stable conditions the separated refractive term agrees with the mast to 0.6 arcseconds RMS. This is sufficient to decide, for any observation of the class considered here, which of the two terms accounts for the observed elevation, and by what margin.

5 Application

The instrument was applied on 11 October 2025 to a specific contested observation: a structure of known height at 11.6 km, photographed from 1.6 m above the waterline, whose visible portion had been offered in support of two incompatible conclusions.

Conditions on the day were stable. The separated refractive term was $k = 0.11 \pm 0.01$. For an eye height of 1.6 m the horizon lies at 4.52 km, and the obscuration of a target at 11.6 km is 3.94 m in the absence of refraction. The measured k gives an effective radius of 7 158 km and predicts 3.24 m. The observed obscuration was 3.2 m, which agrees with the prediction to within the residual in Table 1 and is incompatible with the unrefracted figure at that residual.

6 Discussion

The measurement is decisive to the precision stated, and the precision stated is more than the question requires. We record it in the expectation that it will be of use.

The observation was contested at the time of measurement and remains contested. The Society's programme does not, at present, address this.

References

[1] C. Baak, "Over de meetbaarheid van het geschil," *Verhandelingen van het Genootschap voor Fijne Metingen te Delft*, deel XXII (1887), pp. 143–191.